Rudyard Kipling Primary School and Nursery

Maths Progression in Calculation Policy

RUDYARD KIPLING
Primary School & Nursery,
Woodingdean

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Number – number and place value		
Year 1	**Year 2**	**Year 3**
Essential	**Essential**	**Essential**
• read and write numbers from 1 to 20 in numerals and words. • count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number • count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens • given a number, identify one more and one less • identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least	• count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward • identify, represent and estimate numbers using different representations, including the number line • Reason about the location of any two-digit number in the linear number system including identifying the previous and next multiple of 10. • compare and order numbers from 0 up to 100; use <, > and = signs • read and write numbers to at least 100 in numerals and in words • use place value and number facts to solve problems. • Recognise the place value of each digit in two-digit numbers, and compose and decompose two-digit numbers using standard and non-standard partitioning.	• count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number • recognise the place value of each digit in a three-digit number (hundreds, tens, ones) • compare and order numbers up to 1000 • identify, represent and estimate numbers using different representations • read and write numbers up to 1000 in numerals • solve number problems and practical problems involving these ideas. • Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10. • Reason about the location of any three-digit number in the linear number system, including identifying the previous and next multiple of 100 and 10. • Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.
Greater depth	**Greater depth**	**Greater depth**
• *Memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 − 7 = 9; 7 = 16 − 9).* • *Realise the effect of adding or subtracting 0.* • *Establish addition and subtraction as related operations.* • *Combine and increase numbers, counting forwards and backwards.* • *Discuss and solve problems in familiar practical contexts, including using quantities.* • *Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.*	• *Extend their understanding of the language of addition and subtraction to include sum and difference.* • *Practise addition and subtraction to 20 is fluent in deriving facts such as using 13 + 7 = 20; 20 − 17 = 3 and 20 = 17 + 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30.* • *Check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition.* • *Record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.* • *14 + __ = 15 + 27 Use the inverse to work out difficult problems.* •	• *Practise solving varied addition and subtraction questions.* • *For mental calculations with two-digit numbers, the answers could exceed 100.* • *use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent*

Number – number and place value		
Year 4	**Year 5**	**Year 6**
Essential	**Essential**	**Essential**
• count in multiples of 6, 7, 9, 25 and 1000 • find 1000 more or less than a given number • recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) • order and compare numbers beyond 1000 • identify, represent and estimate numbers using different representations • round any number to the nearest 10, 100 or 1000 • solve number and practical problems that involve all of the above and with increasingly large positive numbers • Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts. • negative numbers • read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.	• read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit • count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 • interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero • round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 • solve number problems and practical problems that involve all of the above • recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents *(from Fractions – including decimals and percentages)* • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 • read Roman numerals to 1000 (M) and recognise years written in Roman numerals. • round decimals with two decimal places to the nearest whole number and to one decimal place	• read, write, order and compare numbers up to 10 000 000 and determine the value of each digit • identify the value of each digit in numbers given to three decimal places (*from Fractions (including decimals and percentages)* • multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places (*from Fractions (including decimals and percentages)* • round any whole number to a required degree of accuracy • use negative numbers in context • solve number and practical problems that involve all of the above. • solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate (*from Measurement*) • use, read, write and convert between standard units, converting measurements of length, mass, and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places (from Measurement) • Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts • use negative numbers in context, and calculate intervals across zero
Greater depth	**Greater depth**	**Greater depth**
• *continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency*	• *practise using the formal written methods of* • *columnar addition with increasingly large numbers to aid fluency.* • *practise mental calculations with increasingly large numbers, e.g., 12,462 – 2,300 = 10,162).* • *introduce the language of algebra as a means of solving a variety of problems* • *mentally add and subtract tenths, and one-digit whole numbers and tenths.* • *practise adding and subtracting decimals, including a mix of whole numbers and decimals, with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).* • *Pupils should go beyond the measurement and money models of decimals to solving puzzle*	• *practise addition for larger numbers, using the formal written methods of columnar addition* • *undertake mental calculations with increasingly large numbers and more complex calculations.* • *round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures.* • *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.* • *Understand common factors can be related to finding equivalent fractions* • *develop the language of algebra as a means of solving a variety of problems*

Number – Addition and Subtraction		
Year 1	**Year 2**	**Year 3**
Essential	**Essential**	**Essential**
• read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs • develop fluency in addition and subtraction facts within 10. • compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers. • add and subtract one-digit and two-digit numbers to 20, including zero • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations • represent and use number bonds and related subtraction facts within 20 • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.	• secure fluency in addition and subtraction facts within 10 through continued practice. • solve problems with addition and subtraction: • using concrete objects and pictorial representations, including those involving numbers, quantities and measures • applying their increasing knowledge of mental and written methods • recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally including: ○ a two-digit number and ones ○ a two-digit number and tens ○ two two-digit numbers ○ adding three one-digit numbers • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. • Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number.	• add and subtract numbers mentally, using formal written methods of columnar addition and subtraction including: ○ a three-digit number and ones ○ a three-digit number and tens ○ a three-digit number and hundreds • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value • Secure fluency in addition and subtraction facts that bridge 10, through continued practice. • Calculate complements to 100, for example: 46 + ? = 100 • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
Greater Depth	**Greater Depth**	**Greater Depth**
• *memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 − 7 = 9; 7 = 16 − 9).* • *realise the effect of adding or subtracting 0.* • *establish addition and subtraction as related operations.* • *combine and increase numbers, counting forwards and backwards.* • *discuss and solve problems in familiar practical contexts, including using quantities.* • *problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.*	• *extend their understanding of the language of addition and subtraction to include sum and difference.* • *practise addition and subtraction to 20 is fluent in deriving facts such as using 13 + 7 = 20; 20 − 17 = 3 and 20 = 17 + 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30.* • *check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition.* • *record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.* • *14 + __ = 15 + 27 Use the inverse to work out difficult problems.*	• *practise solving varied addition and subtraction questions.* • *for mental calculations with two-digit numbers, the answers could exceed 100.* • *use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent*

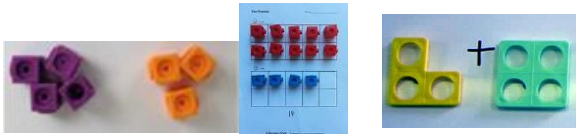
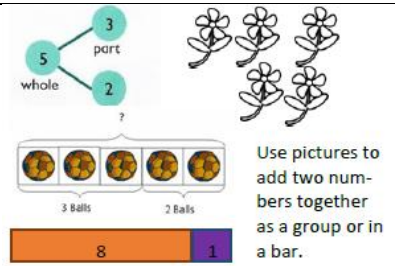
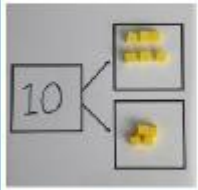
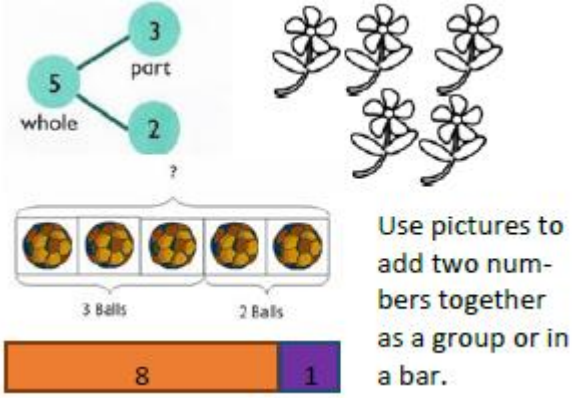
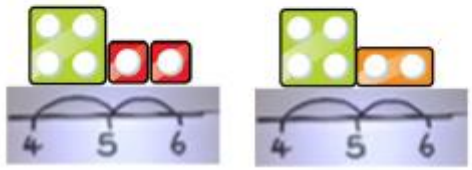
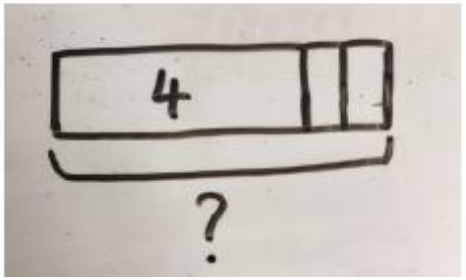
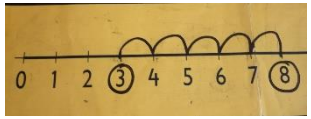
Number – Addition and Subtraction		
Year 4	**Year 5**	**Year 6**
Essential	**Essential**	**Essential**
• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	• add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	• Understand that 2 numbers can be related additively or multiplicatively and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number). • Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding. • perform mental calculations, including with mixed operations and large numbers • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. • find pairs of numbers that satisfy an equation with two unknowns (*from Algebra*)
Greater Depth	**Greater Depth**	**Greater Depth**
• *continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency*	• *practise using the formal written methods of* • *columnar addition with increasingly large numbers to aid fluency.* • *practise mental calculations with increasingly large numbers, e.g., 12,462 – 2,300 = 10,162).* • *introduce the language of algebra as a means of solving a variety of problems* • *mentally add and subtract tenths, and one-digit whole numbers and tenths.* • *practise adding and subtracting decimals, including a mix of whole numbers and decimals, with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).* • *Pupils should go beyond the measurement and money models of decimals to solving puzzle*	• *practise addition for larger numbers, using the formal written methods of columnar addition* • *undertake mental calculations with increasingly large numbers and more complex calculations.* • *round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures.* • *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.* • *Understand common factors can be related to finding equivalent fractions* • *develop the language of algebra as a means of solving a variety of problems*

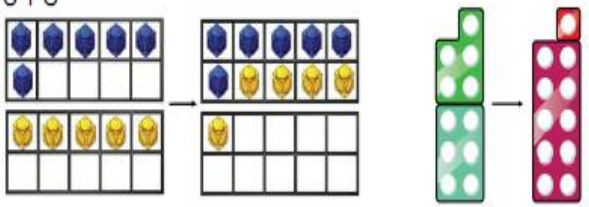
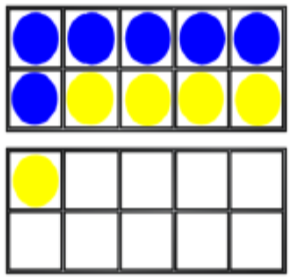
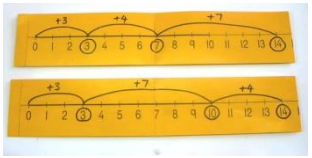
Multiplication and Division		
Year 1	Year 2	Year 3
Essential	**Essential**	**Essential**
• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects pictorial representations and arrays with the support of the teacher. • recognise, find and name a half as one of two equal parts *(from Fractions)* • recognise, find and name a quarter as one of four equal parts of an object, shape or quantity *(from Fractions)*	• recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. • relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).	• recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including any or *teen numbers* times one-digit numbers, using mental and progressing to written methods • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling *by 10* and correspondence problems in which n objects are connected to m objects
Greater Depth	**Greater Depth**	**Greater Depth**
through grouping and sharing small quantities, pupils begin to understand: *multiplication and division;* *doubling numbers and quantities;* *finding simple fractions of objects,* *numbers and quantities* • *make connections between arrays, number patterns, and counting in twos, fives and tens.*	• *use a variety of language to describe multiplication* • *practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other.* • *connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face.* • *begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.* • *Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition.* • *begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40).* • *use inverse relations to develop reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).,* • *use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).*	• *use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.* • *use larger numbers to at least 1,000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146 = 100 + 40 + 6, 146 = 130 +16).* • *use a variety of representations, including those related to measure, pupils continue to count in 1s, 10s and 100s, so that they become fluent in the order and place value of numbers to 1,000.* • *develop efficient mental methods, for example, using commutativity and associativity and multiplication and division facts to derive related facts* • *develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.* • *solve problems in contexts, deciding which of the four operations to use and why. Include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects* • *Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)*

Multiplication and Division		
Year 4	Year 5	Year 6
Essential	**Essential**	**Essential**
• recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. • Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size • Solve division problems, with two-digit dividends and one-digit divisors, that involve remainders, for example: 74 ÷ 9 = 8 r 2 and interpret remainders appropriately according to the context. • Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100).	• identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • Apply place value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth) • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • Understand that 2 numbers can be related additively or multiplicatively and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number). • Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding. • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. • find pairs of numbers that satisfy an equation with two unknowns (*from Algebra*) • solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts (*from Ratio and proportion*)
Greater Depth	**Greater Depth**	**Greater Depth**
• *continue to practise recalling and using multiplication tables and related division facts to aid fluency.* • *practise mental methods and extend this to 3-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).* • *practise to become fluent in the formal written method of short multiplication and short division with exact answers* • *write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)).* • *combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.* • *solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions*	• *practise and extend their use of the formal written methods of short multiplication and short division.* • *apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.* • *use and understand the terms factor, multiple and prime, square and cube numbers.* • *interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = 98/4 = 24 r 2 = 241/2 = 24.5 ≈ 25).* • *use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres.*	• *practise multiplication and division for larger numbers, using the formal written methods of short and long multiplication and division.* • *undertake mental calculations with increasingly large numbers and more complex calculations.* • *continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.* • *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.* • *understand common factors can be related to finding equivalent fractions* • *develop the connection made between multiplication and division with fractions, decimals, percentages and ratio* • *develop the language of algebra as a means of solving a variety of problems*

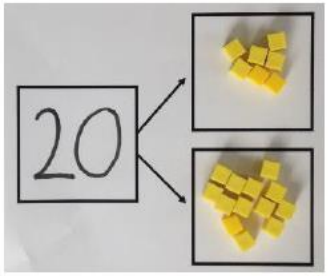
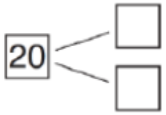
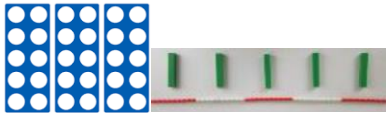
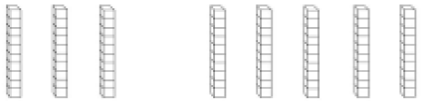
such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children *Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)*	• *introduce the language of algebra as a means of solving a variety of problems* • *distributivity can be expressed as a(b + c) = ab + ac.* • *understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).* • *use and explain the equals sign to indicate equivalence, including in missing number problems (for example 13 + 24 = 12 + 25; 33 = 5 x ?)* *Key language – dividend (27) divided by the divisor (9) that will equal the quotient (3)*	• *multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers.* • *multiply decimals by whole numbers, starting with the simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money.* • *practise division for larger numbers, using the formal written methods of short and long division* • *explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.* • *Understand common factors can be related to finding equivalent fractions* • *develop the connection made between multiplication and division with fractions, decimals, percentages and ratio* • *explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375).* • *For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context.* • *introduce to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money.* • *recognise division calculations as the inverse of multiplication.* • *develop the skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations, including rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.*

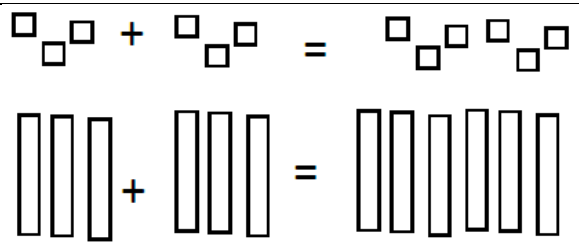
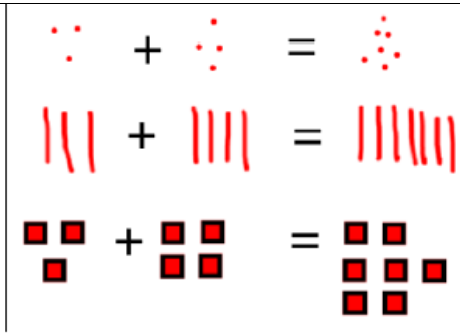
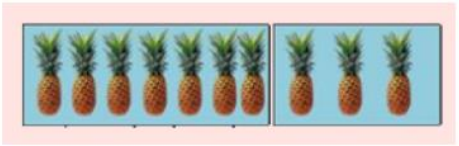
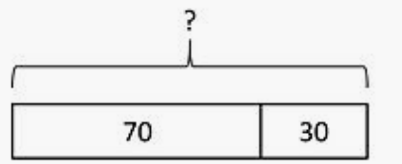
Reception to Year 1 – CPA Approach: Addition

Objective and Strategy	Concrete	Pictorial	Abstract
Reception **Addition & subtraction**	Using real-life addition stories and scenarios to support understanding (e.g – There are 5 people on the bus. 3 more get on. How many now?  The children will start to use five frames and then build up to 10 frames.	 Use pictures to add two numbers together as a group or in a bar.	8 + 1 = 9 9 – 1 = 8
Reception **Combining two parts to make a whole part: part-whole model.**	  Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	 Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Counting on using number lines.	Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	Using the abstract number line to count on in 1s 

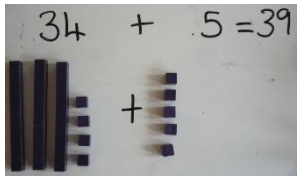
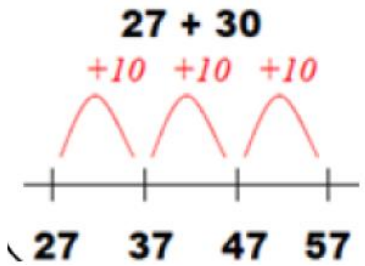
Regrouping to make 10	Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Use number line and Numicon to show equality in addition, and addition can be done in any order. 3 + 4 + 7 = 3 + 7 + 4 

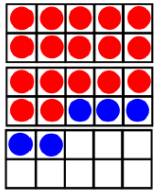
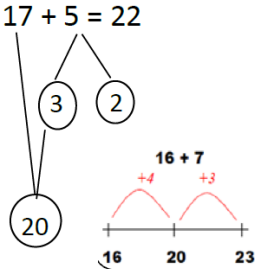
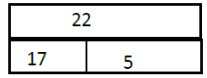
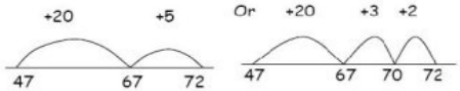
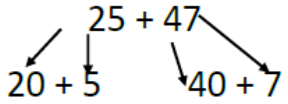
Year 2 - CPA Approach: Addition

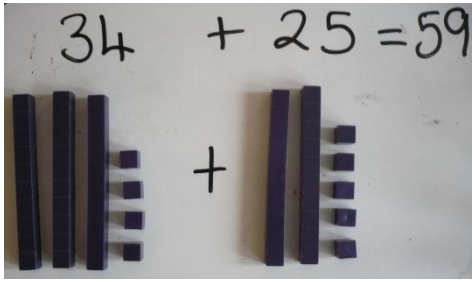
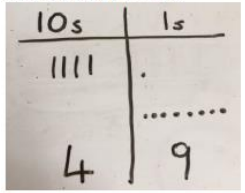
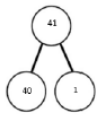
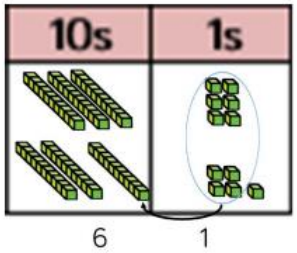
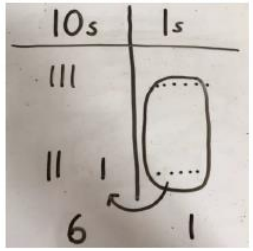
Objective and Strategy	Concrete	Pictorial	Abstract
Use known facts: part, part whole.	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Adding multiples of ten	**30 + 20 = 50 50 = 20 + 30** 	 3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60

Adding multiplies of ten: Use known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Adding multiples of ten: Bar model		 7 + 3 = 10	 70 + 30 = 100

Year 2 - CPA Approach: Addition

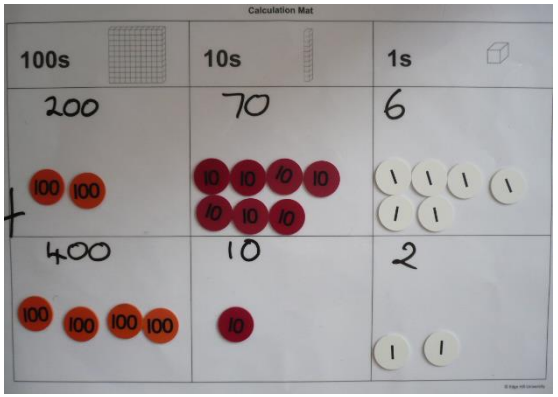
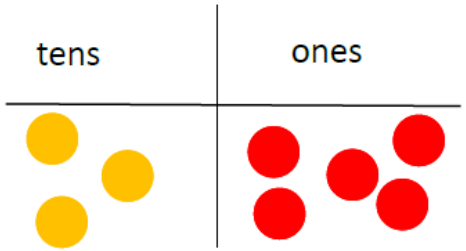
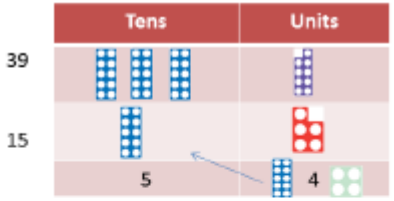
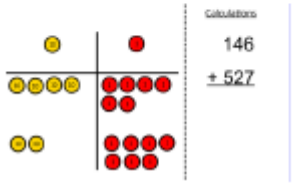
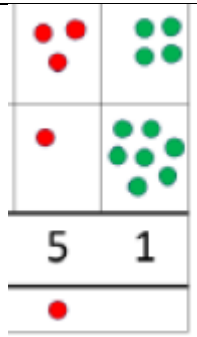
Objective and strategy	Concrete	Pictorial	Abstract
Add a two digit number and tens.	 **Explore why the ones digit doesn't change.**		27 + 10 = 37 27 + 20 = 47 27 + □ = 57

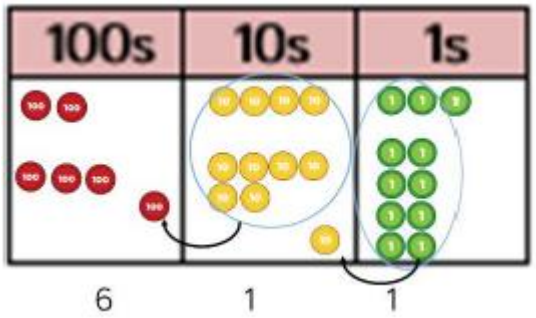
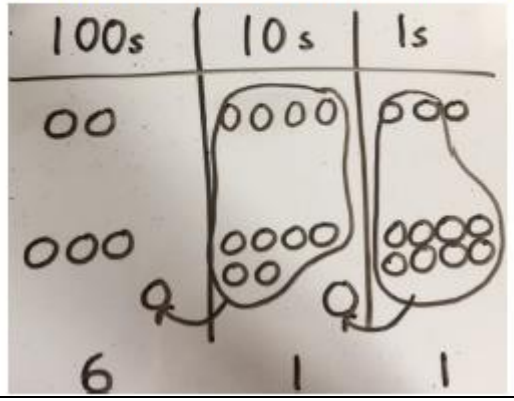
		Differentiation: either use real numbers lines or children to draw their own empty number lines.	
Add a two digit number and ones. **The 'making ten first' strategy.**	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a two 2 digit numbers.	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 57 + 41 = 98 50 + 40 = 90 7 + 1 = 8 98 Partitioning

Add a two 2 digit numbers.		Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	Use the formal method only if children are secure with their mental strategies. 41 + 8  1 + 8 = 9 40 + 9 = 49 4 1 + 8 4 9
Continue to develop partitioning and place value: TO + TO	36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Only move onto the formal method if children are secure with their mental strategies of adding TO + TO. Looking for ways to make 10. 36 + 25 = 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 +25 61 1 Formal method:

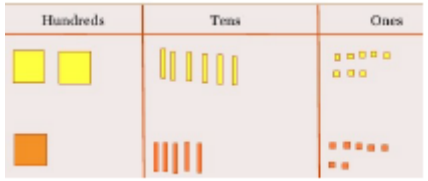
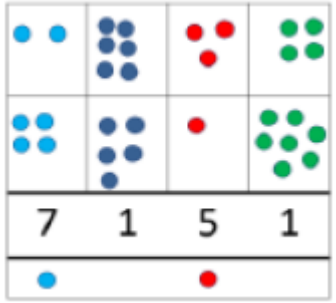
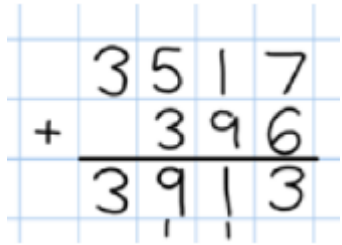
Year 3 – CPA Approach: Addition

Objective and strategy	Concrete	Pictorial	Abstract

Add numbers up to 3 digits using a formal written method **Column Addition – No regrouping**		Children move to drawing the counters using a tens and one frame. 	$$\begin{array}{r} 223 \\ +\ 114 \\ \hline 337 \end{array}$$ Add the ones first, then the tens, then the hundreds.
Add numbers up to 3 digits using a formal written method **Column Addition – Regrouping**	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	$$\begin{array}{rcrl} 20 & + & 5 & \\ 40 & + & 8 & \\ \hline 60 & + & 13 & = 73 \end{array}$$ Start by partitioning the numbers before formal column to show the exchange. $$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 11 \end{array}$$

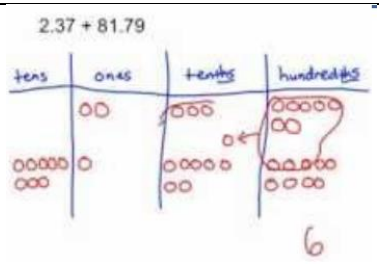
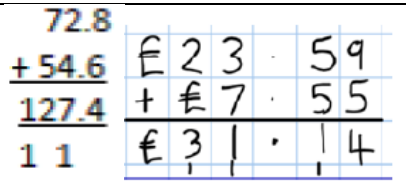
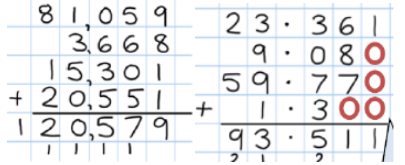
Add numbers up to 3 digits using a formal written method		Chidren to represent the counters in a place value chart, circling when they make an exchange. 	243 +368 611 1 1

Year 4 - CPA

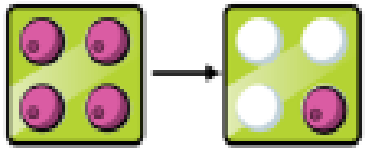
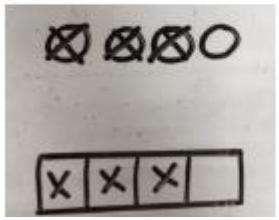
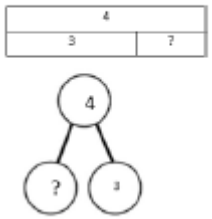
Objective and strategy	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits using the formal written methods of column addition.	Children continue to use dienes or place value counters to add, exchanging tens ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 		 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.

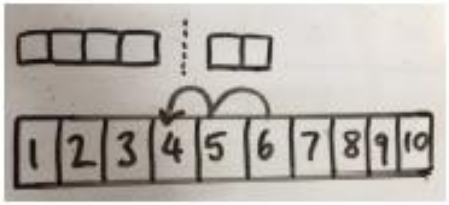
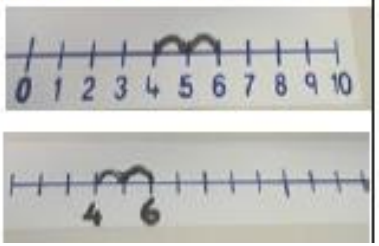
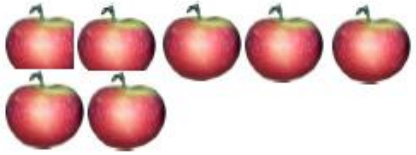
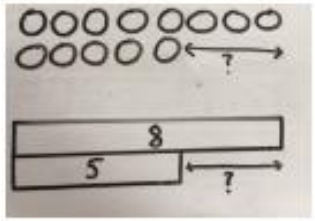
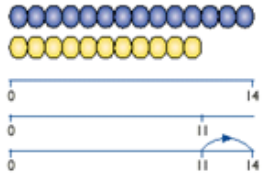
Year 5 & 6 - CPA

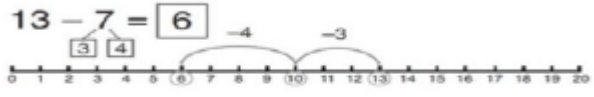
Objective and strategy	Concrete	Pictorial	Abstract

Year 5 **Add numbers with more than 4 digits.** **Add decimals with 2 decimals places, including money.**	As year 4  Introduce decimal place value counters and model exchange for addition.	 	
Year 6 **Add numbers of increasing complexity** **Including adding money, measure and decimals with different numbers of decimal points.**			 Insert zeros for place holders.

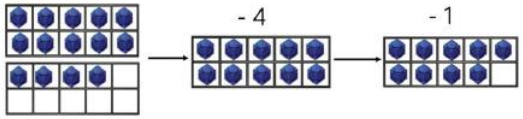
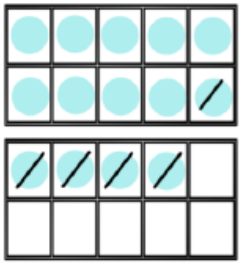
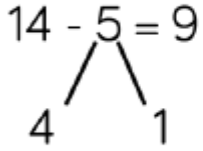
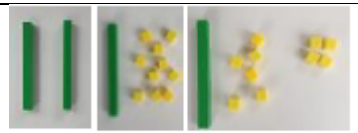
Reception - Year 1 – CPA Approach: Subtraction

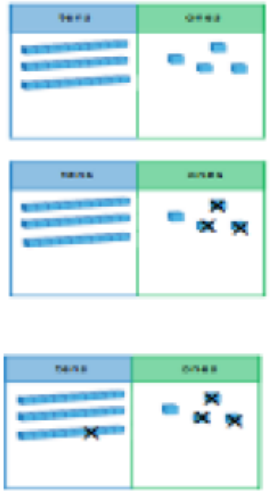
Objective & Strategy	Concrete	Pictorial	Abstract
Subtract a one digit number.	Physically taking away and removing objects from a whole. 4 - 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 - 3 

Counting back	Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 
Bar model	 5—2 = 3		8 | 2 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2
Find the difference	Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5.  10-4=6 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	 The difference between 11 and 14 is 3. 14 – 11 = 3 11 + □ = 14 Find the difference between 8 and 5. 8 – 5, the difference is □ Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference.

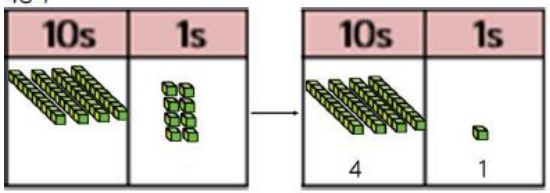
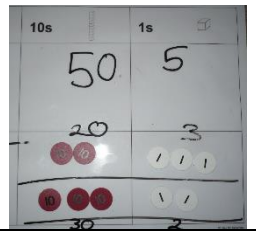
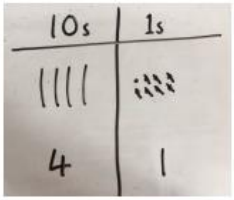
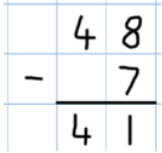
Subtraction using the 'making 10' strategy.	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?

Year 2 – CPA Approach: Subtraction

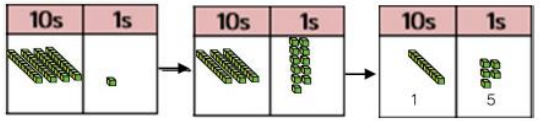
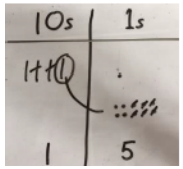
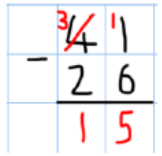
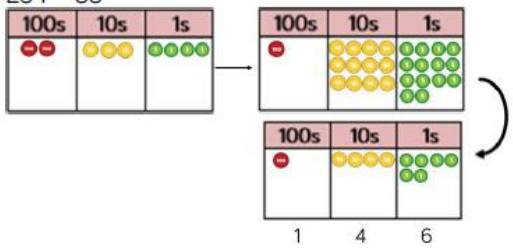
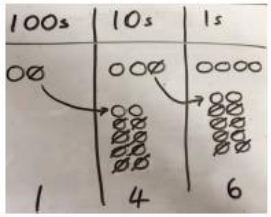
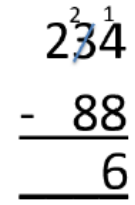
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting using the 'making ten first' strategy.	Making 10 using ten frames. 14 – 5  	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 20 – 4 =	20—4 = 16

Partitioning to subtract without regrouping	34—13 = 21 Use Dienes to show how to partition the number when subtracting without regrouping.		Children draw representations of Dienes and cross off.  43—21 = 22	55 - 23 = 50 5 - 20 3 30 2 = 32

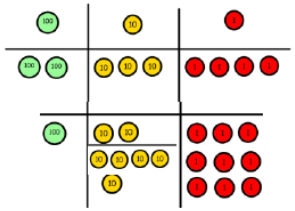
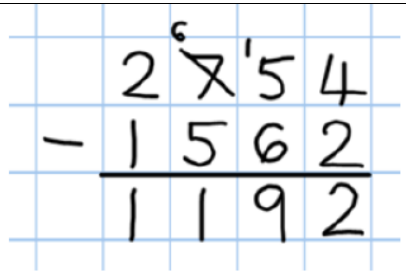
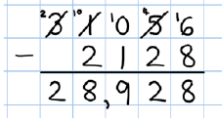
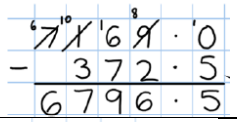
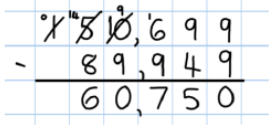
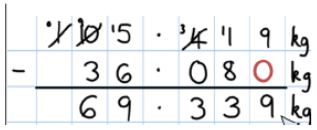
Year 2/3 – CPA Approach: Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Subtract numbers including: **A two digit and ones** **A two digit and tens** **Two two digits**	Column method using base 10. 48-7   	Children to represent the base 10 pictorially.  	Column method or children could count back 7. 

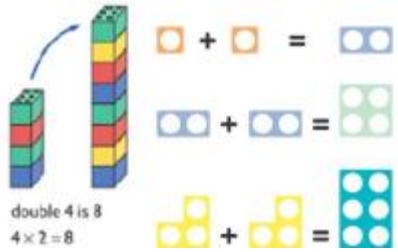
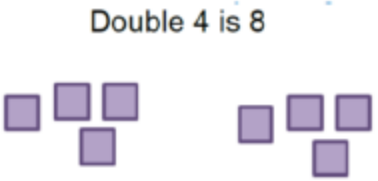
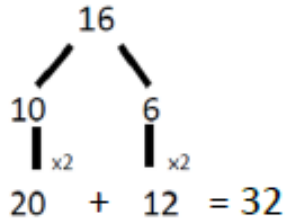
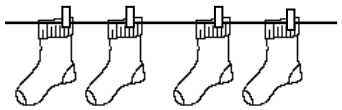
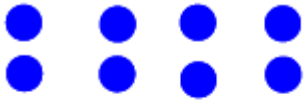
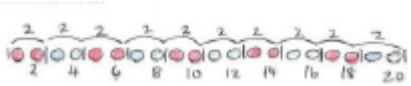
Year 3 – CPA Approach: Subtraction

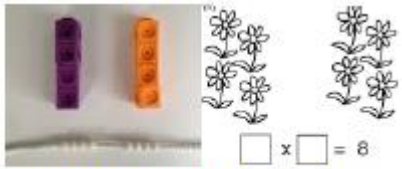
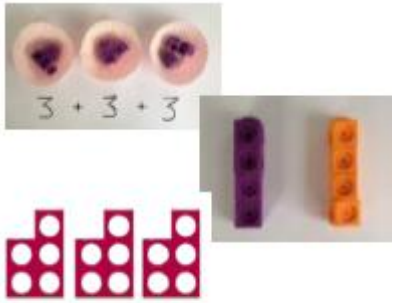
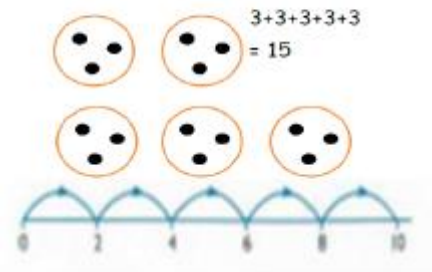
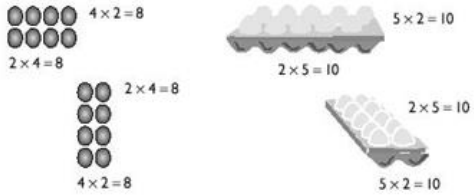
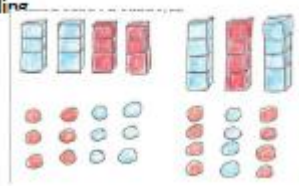
Objective & Strategy	Concrete	Pictorial	Abstract
Subtract numbers with up to three digits using a formal written method. **Column subtraction - Regrouping**	Column method using base 10 and having to exchange. 41 – 26  	Represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Subtract numbers with up to three digits using a formal written method. **Column subtraction – Regrouping**	Column method using place value counters. 234 – 88  	Represent the place value counters pictorially; remembering to show what has been exchanged.  	Formal colum method. Children must understand what has happened when they have crossed out digits. 

Year 4, 5 & 6 – CPA Approach: Subtraction

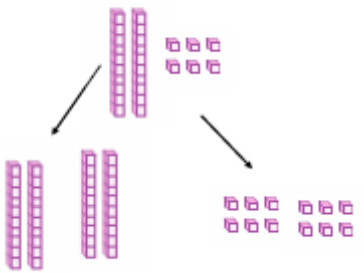
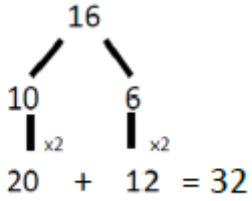
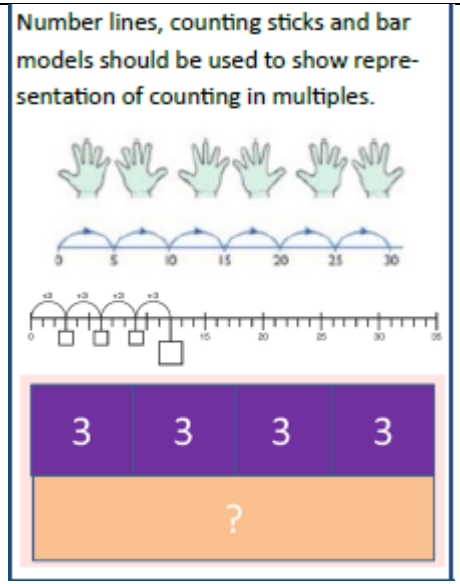
Objective & Strategy	Concrete	Pictorial	Abstract
Year 4 **Subtract tens and ones.** **Subtract up to 4 digits.** *Introduce decimal subtraction through context of money.*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw place value counters and show their regrouping – see Y3.	 Use the phrase 'take and make' for exchange
Year 5 **Subtract with at least 4 digits, including money and measures.** *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal.*			 Use zeros for placeholders. 
Year 6 **Subtract with increasingly large and more complex numbers and decimal values.**			 

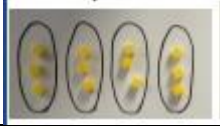
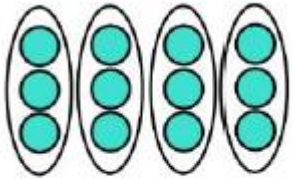
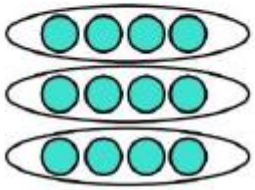
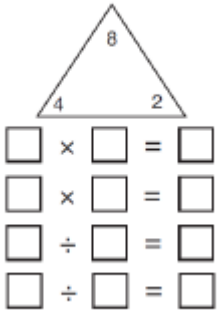
Reception - Year 1 – CPA Approach: Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.     10p + 10p + 10p = 30p 3 lots of 10p = 30p 10, 20, 30 Count the number of coins, 3 coins = 30p How many coins to make 30p?	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

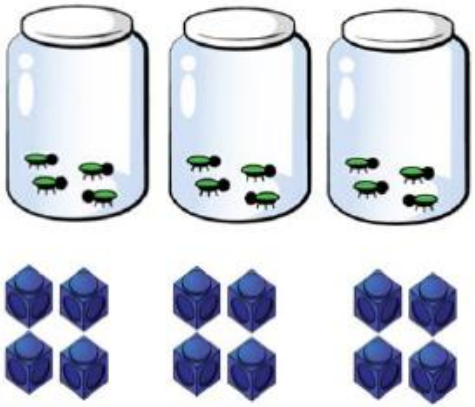
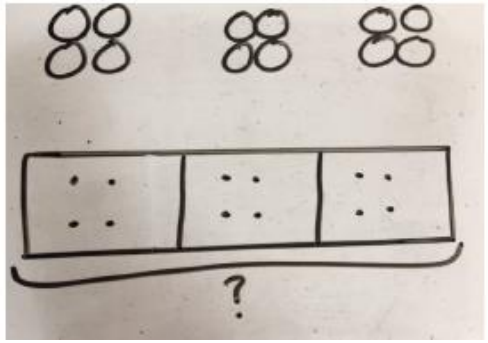
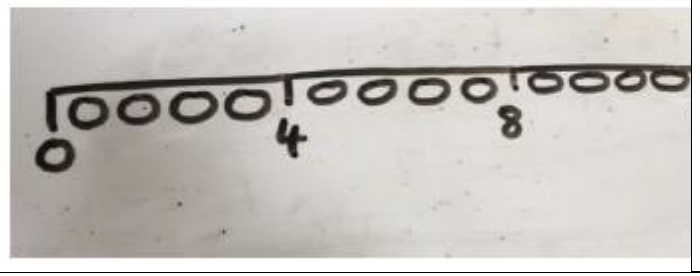
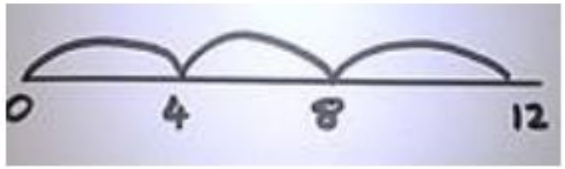
Making equal groups and counting the total.	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	 Use real life arrays – "2 lots of 4 is ___"	Draw representations of arrays to show under-standing 	3 x 2 = 6 2 x 5 = 10

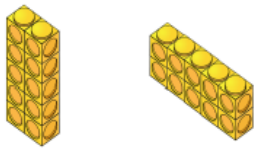
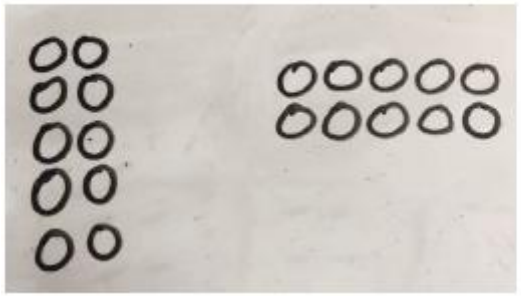
Year 2 – CPA Approach: Multiplication

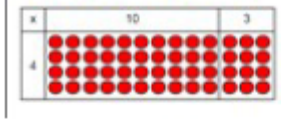
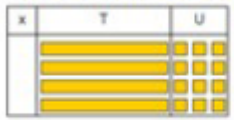
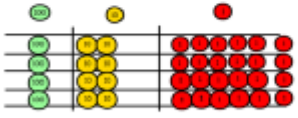
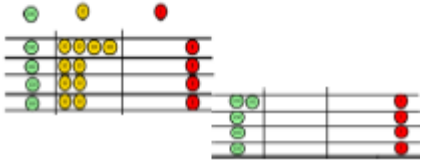
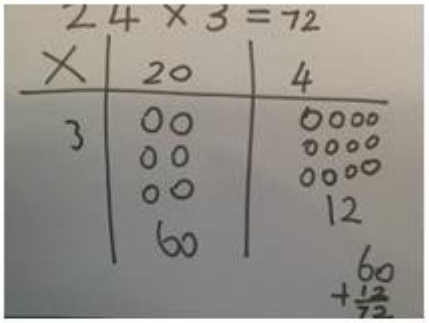
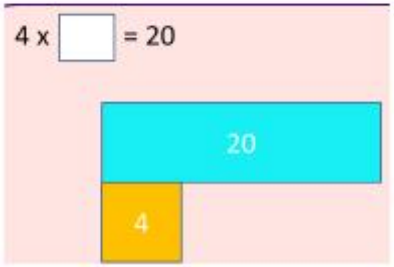
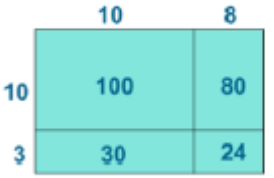
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  16 10 x2 6 x2 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0. **(Repeated addition)**	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   ?	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Multiplication is commutative.	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 x 4 12 = 4 x 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse. *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Year 3 – CPA Approach: Multiplication

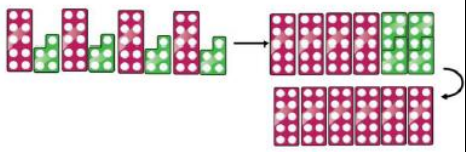
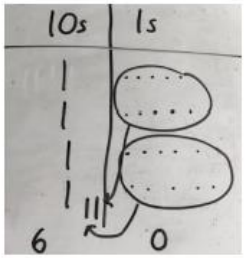
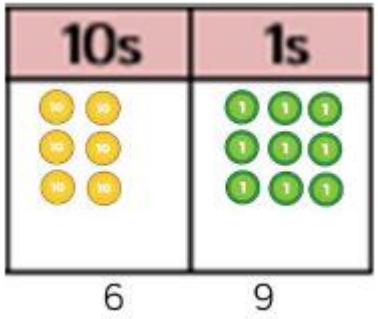
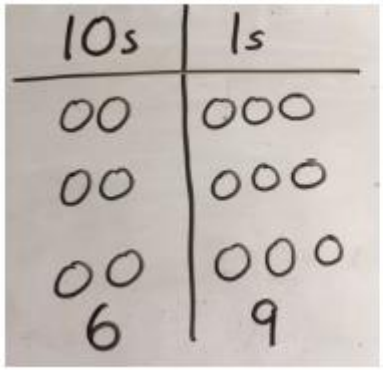
Objective & Strategy	Concrete	Pictorial	Abstract
Repeated grouping/ Repeated addition	3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each gr 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Repeated grouping/ Repeated addition	3 × 4 	Represent this pictorially alongside a number lin 	Abstract number line showing three jum of four. 3 × 4 = 12 

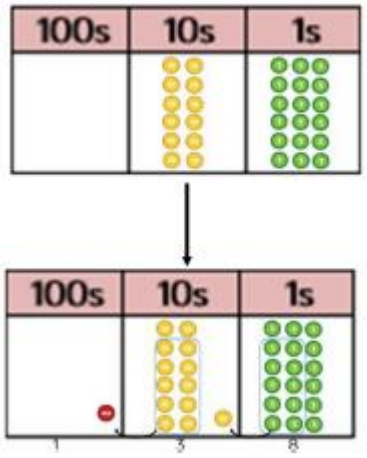
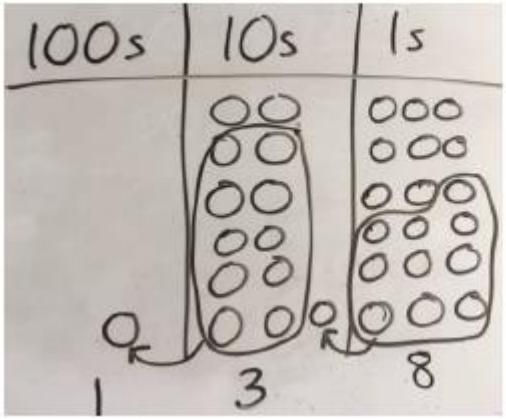
Use arrays to illustrate commutativity.	3 x 5 = 5 x 3  2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to wr range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Grid method	Show the links with arrays to first introduce the grid method  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 

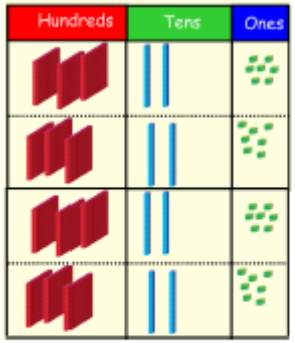
×	30	5
7	210	35

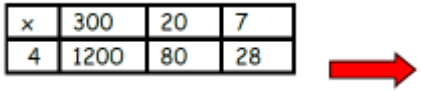
Year 4 – CPA Approach: Multiplication

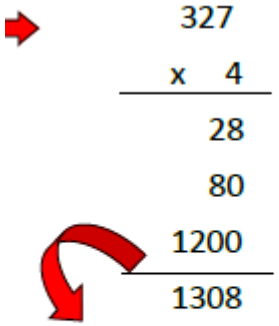
Objective & Strategy	Concrete	Pictorial	Abstract
Partition to multiply.	4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 15 x 5 = 10 x 5 = 50 5 x 5 = 25 75
Multiply a two-digit & three-digit numbers by a one digit number using a formal method.		Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

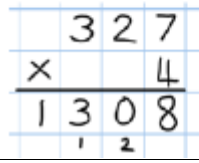
	6 x 23  	Children to represent the counters/base 10, pictoriall e.g. the image below.  	Formal written method 6 x 23 = 23 × 6 138 1 1

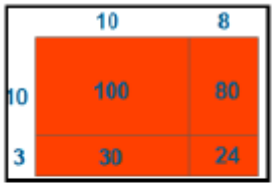
Year 5 & 6 – CPA Approach: Multiplication

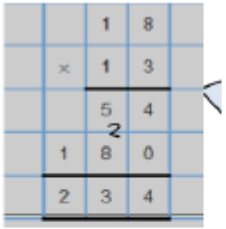
<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column multiplication for 3 and 4 digit numbers x 1 digit.</td>
<td>


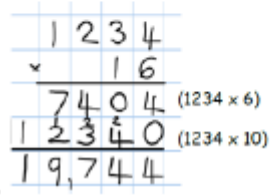
It is important at this stage that they always multiply the ones first.
Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642
</td>
<td>


</td>
<td>


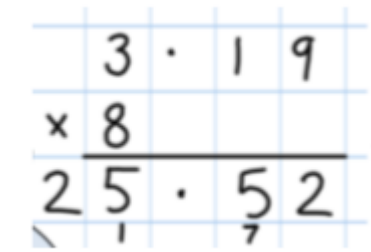


This will lead to a compact method.
</td>
</tr>
<tr>
<td>Column multplication</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</td>
<td>


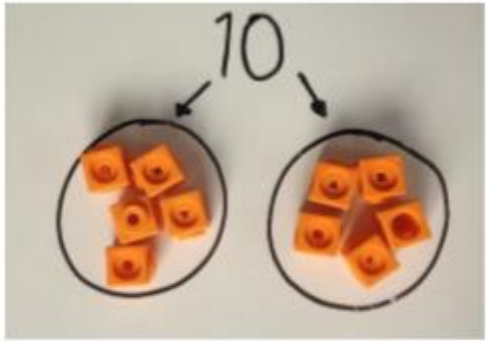
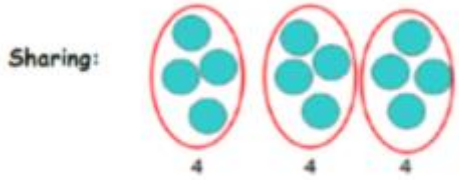
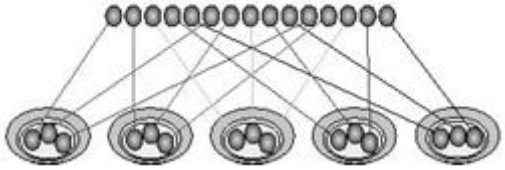
Continue to use bar modelling to support problem solving
</td>
<td>


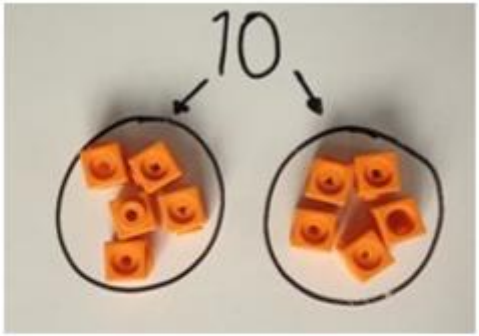
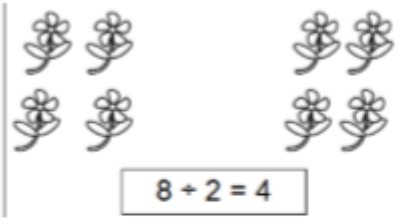
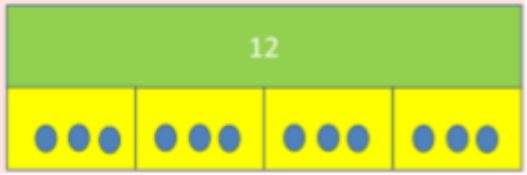
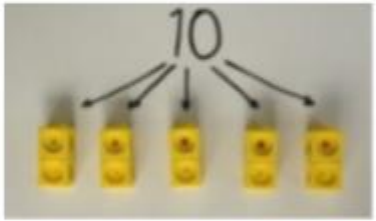
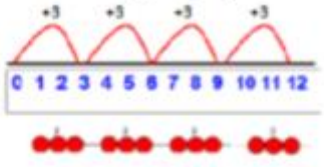
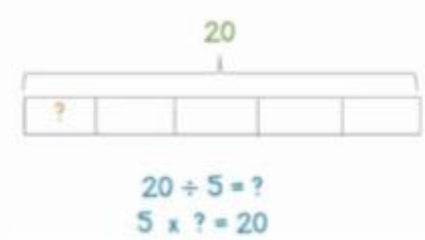
18 x 3 on the first row
(8 x 3 =24, carrying the 2 for 20, then 1 x 3)
18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first


</td>
</tr>
</table>

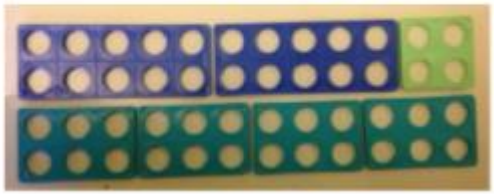
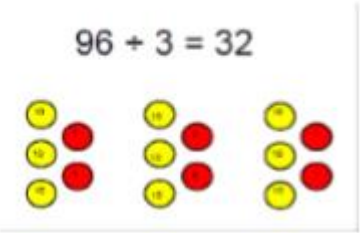
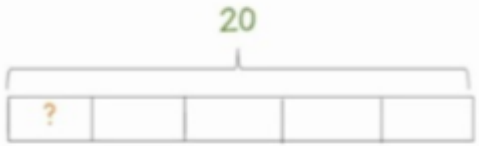
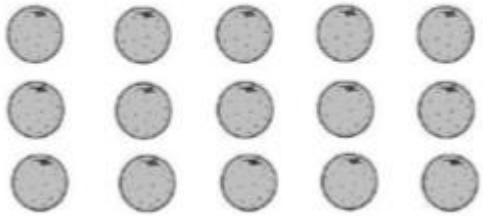
Year 6 **Multiplying decimals up to 2 decimal places by a single digit.**			

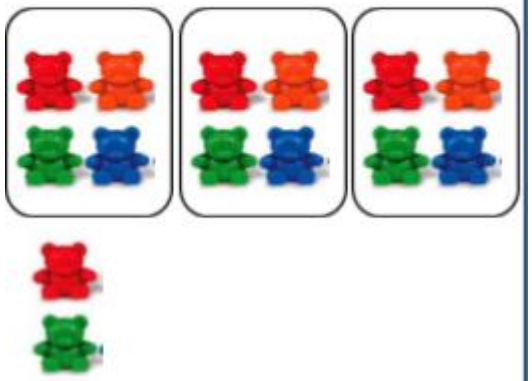
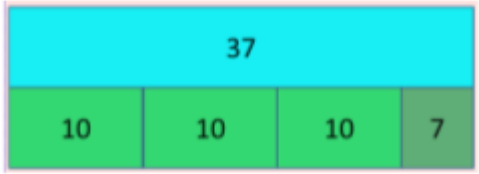
Year 1 & Y2 – CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
Year 1 **Division as sharing**	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4  Sharing: 4 4 4 12 shared between 3 is 4 15 ÷ 5 = 3 15 shared between 5 	12 shared between 3 is 4

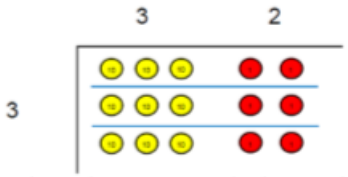
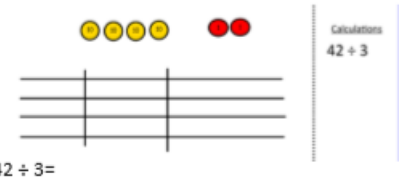
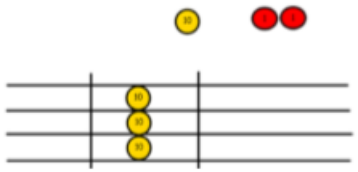
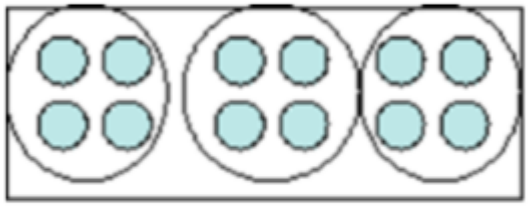
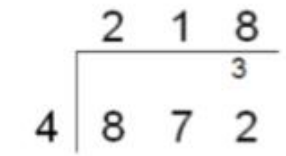
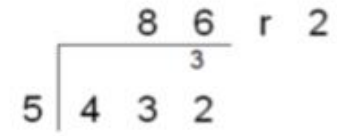
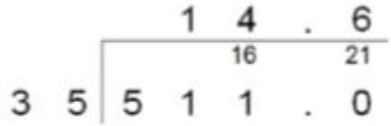
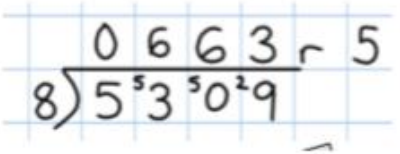
Year 2 **Division as sharing**	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  **12 ÷ 3 = 4** Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Year 3 – CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

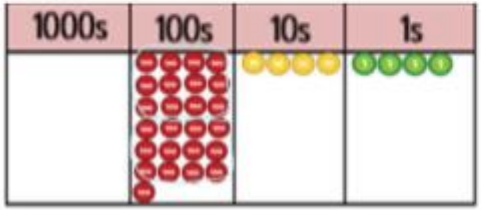
Year 4 & 5– CPA Approach: Division

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. **Short division**	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Year 6 – Long division

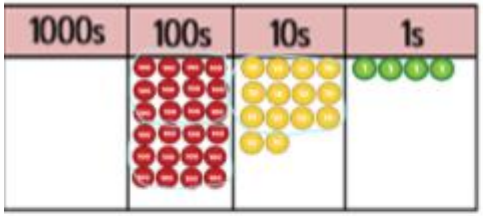
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



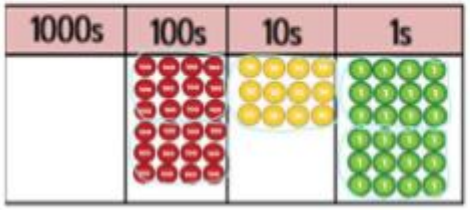
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$



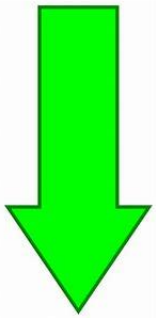
After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$



1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)278	h t o 1 2)278 -2 0	h t o 18 2)278 -2↓ 07
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 13 2)278 -2 07	h t o 13 2)278 -2 07 -6 1	h t o 13 2)278 -2 07 -6 18
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 139 2)278 -2 07 -6 18	h t o 139 2)278 -2 07 -6 18 -18 0	h t o 139 2)278 -2 07 -6 18 -18 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.